SM3H HW11.2 Vectors

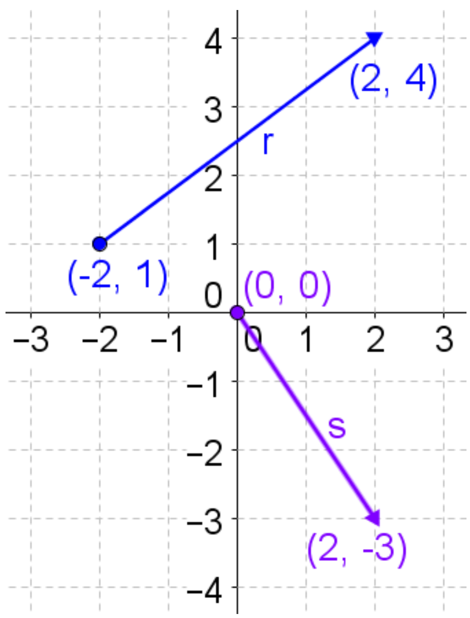
Vocab: Vector: A direction with magnitude, useful for measuring how points in space are related to one another.

Vectors are objects, not numbers; in the diagram, the vector is the arrow.

Vectors have direction. The starting point is called the tail and the ending point is called the head.

The magnitude of a vector is calculated in the same manner that the distance of the hypotenuse of a right triangle is calculated.

A vector in $\mathbb{R}^n$ space is written:
$v =< v_1, v_2, \ldots, v_{n-1}, v_n >$
Where $v_1, v_2, \ldots, v_{n-1}, v_n$ are the components of the vector.



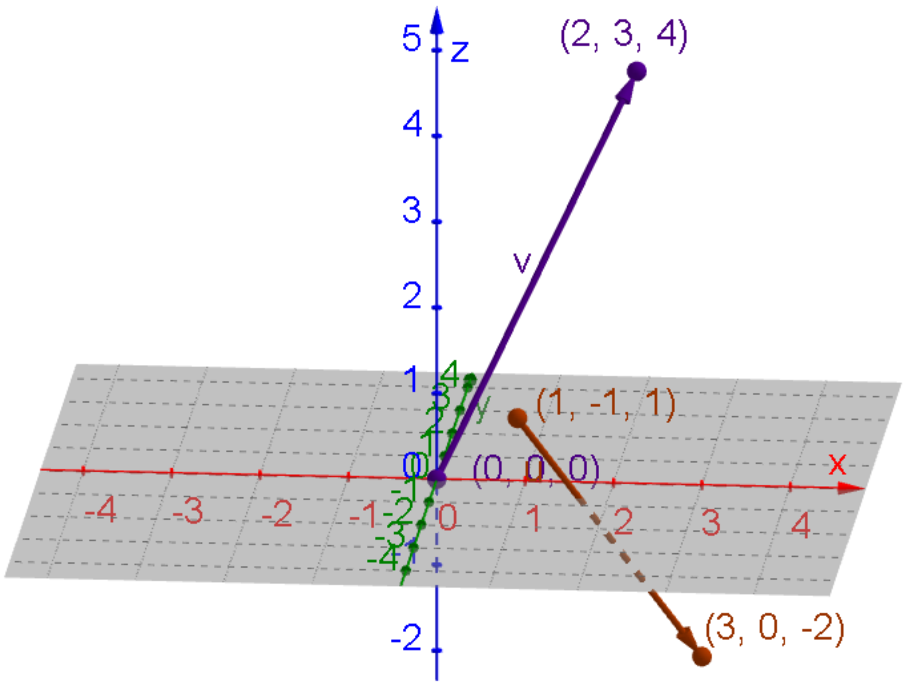
r is a vector in $\mathbb{R}^2$. So, there are 2 components of vector r. To calculate each component, we take the value of the component at the head minus the value of the component at the tail:

$r =< 2 - (-2), 4 - 1 >$
$r =< 4, 3 >$

s is a vector in $\mathbb{R}^2$. So, there are 2 components of vector s.

$s =< 2 - 0, -3 - 0 >$
$s =< 2, -3 >$

Generally, linear vectors in $\mathbb{R}^2$ have the form $v =< run, rise >$.



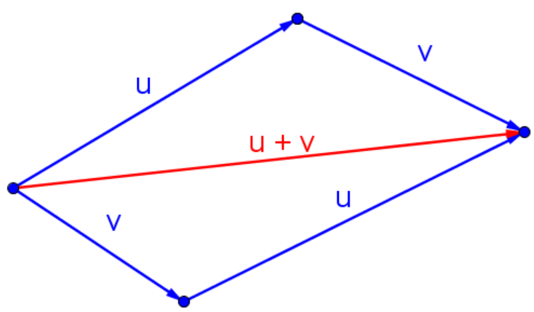
v is a vector in $\mathbb{R}^3$. So, there are 3 components of vector v.

$v =< 2 - 0, 3 - 0, 4 - 0 >$
$v =< 2, 3, 4 >$

u is a vector in $\mathbb{R}^3$. So, there are 3 components of vector u.

$u =< 3 - 1, 0 - (-1), -2 - 1 >$
$u =< 2, 1, -3 >$

Vector Addition

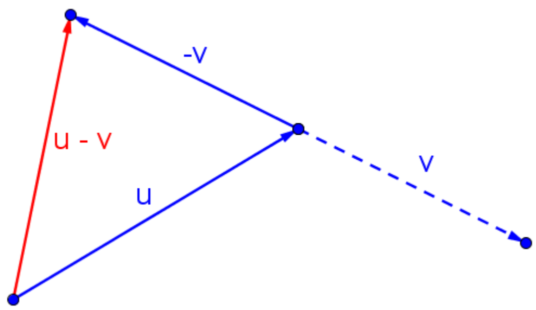


Adding vectors means travelling along one vector and then beginning the second vector where the first one ended. It matters not which vector gets travelled first; their sum will end in the same location.

Example: $u =< 5, 3 >; v =< 4, -2 >$

$u + v =< 5 + 4, 3 + (-2) >$
$u + v =< 9,1 >$

Vector Subtraction



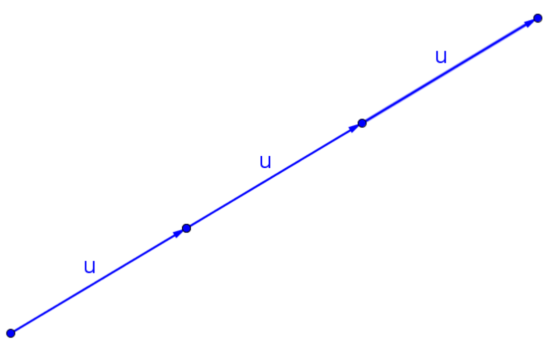
Subtracting vectors means travelling along one vector and then beginning the second vector where the first one ended but following the second vector in reverse order (head to tail instead of tail to head). This is equivalent to negating each component of the subtracted vector and then adding.

Example: $u = < 5, 3 >; v = < 4, -2 >$

$-v = < -4, 2 >$

$u - v = u + (-v)$
$u - v = < 5 + (-4), 3 + 2 >$
$u - v = < 1, 5 >$

Scalar Multiplication



Adding several copies of the same vector can be done by group addition: multiplication. It is important to note that we are not multiplying a vector with another vector. We're multiplying a vector with a number, which is called a scalar. This is equivalent to distributing the scalar over the vector.

Example: $u = < 5, 3 >; k = 3$

$ku = k < 5, 3 >$
$ku = < (3)5, (3), 3 >$
$ku = < 15, 9 >$

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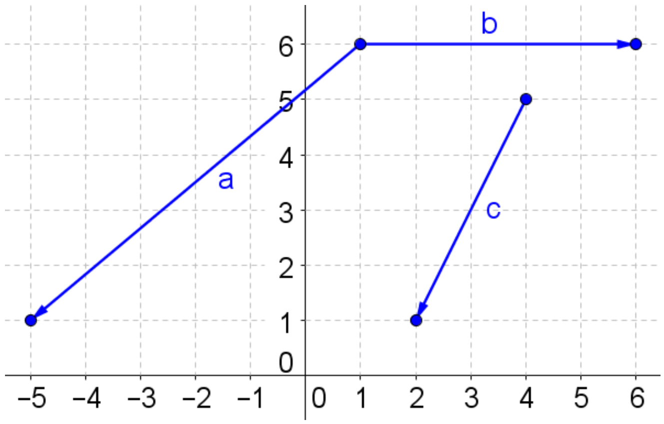


Use the figure for problems 1-12

1) State the components of vector a

2) State the components of vector b

3) State the components of vector c

4) State the components of vector $a+b$

5) State the components of vector $b-c$

6) State the components of vector $c+a$

7) State the components of vector $2a$

8) State the components of vector $-3c$

9) State the components of vector $3(a-b)$

10) State the components of vector $4c+5a$

11) State the components of vector $9b-6a$

12) State the components of vector $2(7b-4c)$

Sketch and label the following vectors with tail of $(0,0)$ on the coordinate axis:

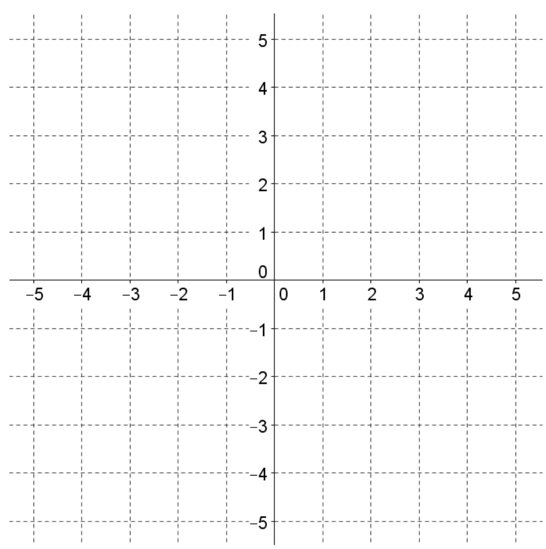
13) $g =< 5, 1 >$

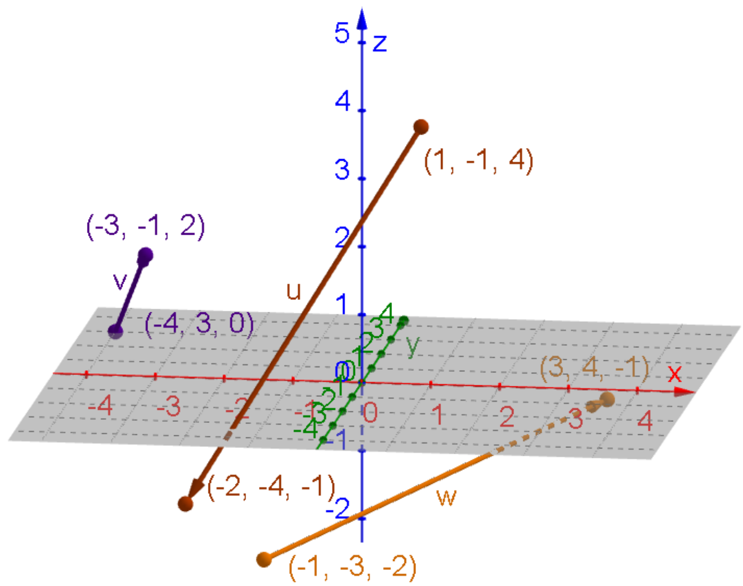
14) $h =< 3, -2 >$

Sketch and label the following vectors on the coordinate axis:

15) $p =< 2, 2 >$; p has tail of $(-4, -3)$

16) $q =< -1, 3 >$; q has head of $(-2,5)$





Use the figure for problems 17-25

17) State the components of vector u

18) State the components of vector v

19) State the components of vector w

20) State the components of vector $u + v$

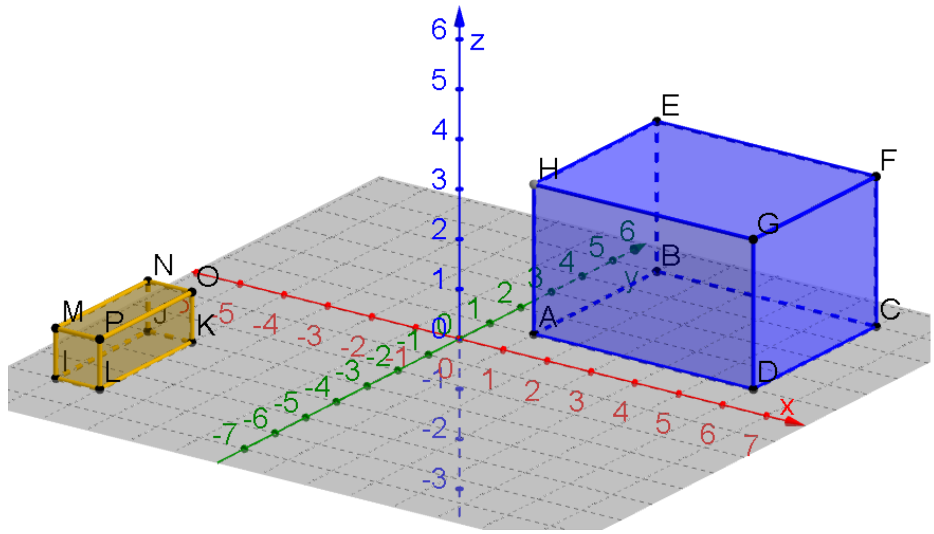
21) State the components of vector $v - w$

22) State the components of vector $u - v + w$

23) State the components of vector $3v$

24) State the components of vector $-5w$

25) State the components of vector $7(w - u)$



Use the figure for questions 26-28

Mr. Wytiaz writes a challenging test for his secondary math 3 class (taught in the large building on the right). Misha, Lila, and Emily double check their answers and work until the end of class. Worried that they will be late for seminary (taught in the small building on the left), they each use an alternate route to try and get to “class” on time.

26) Lila hands her test in, darts into the hallway, and pushes out the window that she secretly detached earlier in the year, in case she had to get out of the building onto the roof in an emergency. She grabs the grappling hook and rope that she stashed on the roof, walks to point H in the figure and loads the hook into the small cannon that her older sister installed. The hook is fired onto the corner of the seminary building (point O) and she rides a makeshift metal hangar on the rope to seminary. What are the components of vector l, which represents Lila’s path along the rope?

27) Misha hands her test in, kicks off the boot she was using to get an elevator pass and takes the elevator (which is at point E) to the secret level one floor underneath the school. She exits the elevator and sprints along the underground passage that goes directly under point L of the seminary building. At that point, she passes the voice-activated check (by reciting her scripture mastery) that drops a ladder down from her seminary class, which she climbs to get to class on time. What are the components of vector m, which represents Misha’s motions along the underground path?

28) Emily hands her test in and glances at the clock. Realizing that she won’t get to seminary on time, she walks to a window at point G and her eyes bright and white. The seminary building begins to tremble and then rises off of the ground! Slowly but steadily the flying seminary building approaches the school, spinning just so that an open window at point M aligns with the school’s window at point G. Emily levitates through both windows, thus entering her seminary class. She smiles at her panicked seminary instructor as the building returns to its original position and takes root. What are the components of vector s, which represents the direct flight path of the window (M) of the seminary building as it approached the window (G) of the school.